Diagnostics for Turbulence in Plasmas

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Introduction to Turbulence and Goals

- Turbulence is a characteristic of a fluid flow that occurs at high Reynold's numbers
- Plasmas can be approximated as fluids because it is a viscous flow travelling at high speeds with electromagnetic forces acting on them

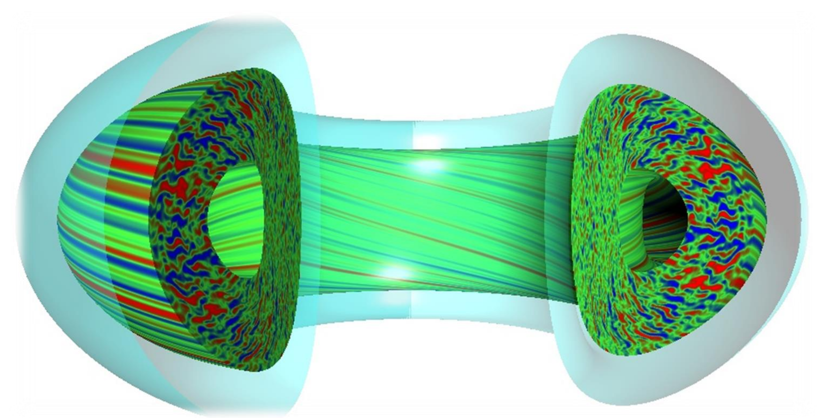


Figure 1: Turbulence within plasmas visualized in the tokamak [3]

- The goal of this project is to develop methods for analyzing turbulence and integrate these methods in the Gkeyll code
- For 2D and 3D simulations, the following codes were developed:
  - Convert data to polar coordinates and plot the power spectrum of the data
  - Visualize the data in 3D
  - Calculate and plot the evolution of kinetic energy in a simulation
  - Calculate and plot the time derivative of the kinetic energy
  - Calculate and plot the enstrophy of a simulation

Governing Equations

- The Euler equations were used to simulate the fluid simulations:

$$\boldsymbol{y} = \begin{bmatrix} \rho \\ \boldsymbol{j} \\ E^t \end{bmatrix} \qquad \boldsymbol{j} = \rho \boldsymbol{u} \qquad E^t = \rho e + \frac{1}{2}\rho u^2 \qquad \boldsymbol{F} = \begin{bmatrix} \boldsymbol{j} \\ \frac{1}{\rho}\boldsymbol{j} \otimes \boldsymbol{j} + p\boldsymbol{I} \\ (E^t + p)\frac{\boldsymbol{j}}{\rho} \end{bmatrix}$$

$$\frac{\partial}{\partial t}(\boldsymbol{y}) + \nabla \cdot \boldsymbol{F} = \begin{bmatrix} 0 \\ \boldsymbol{f} \\ \frac{\boldsymbol{j}}{\rho} \cdot \boldsymbol{f} \end{bmatrix} \qquad \boldsymbol{f} = 0$$

- For the plasma case, the Euler equations above were coupled with Maxwell's equations to solve for the magnetic and electric fields

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0} \qquad \nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{B} = \mu_0(\boldsymbol{J} + \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}) \qquad \boldsymbol{f} = q\boldsymbol{E} + q\boldsymbol{v} \times \boldsymbol{B}$$

Kelvin-Helmholtz

- 2D simulation of a velocity shear in a continuous fluid with $\frac{u_0}{c_s} = 0.4$
- Simulated in the Gkeyll code by creating a velocity shear in the fluid by making the upper half and lower half of the fluid move in opposite directions

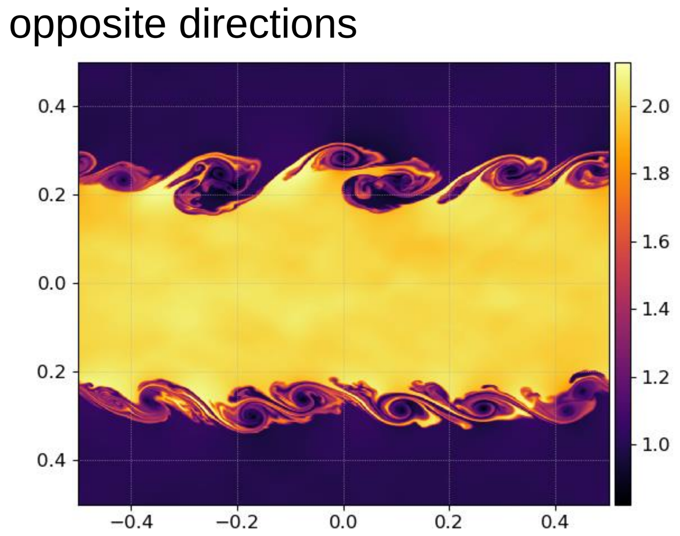


Figure 2: Energy production range of the density of the fluid depicted at time = 1. The eddies are starting to develop and the waves are beginning to interact with each other.

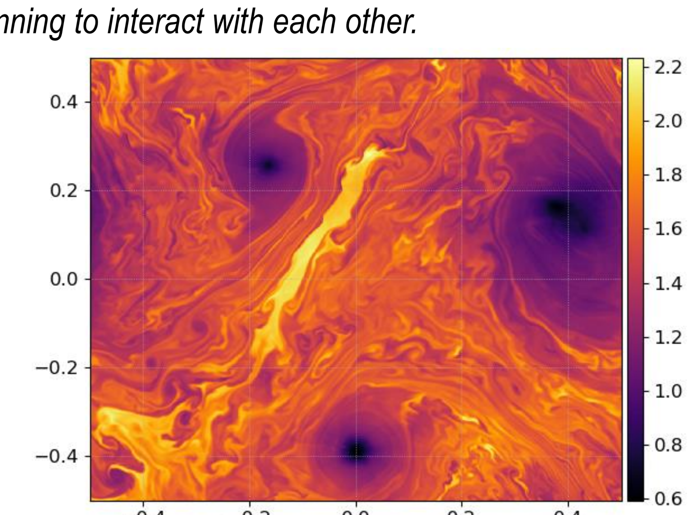


Figure 3: The inertial sub-range of the fluid density depicted at time = 10. Three main vortices are shown in the figure and the flow starts to exhibit turbulent characteristics. The eddies are highly energetic, perturbing the flow.

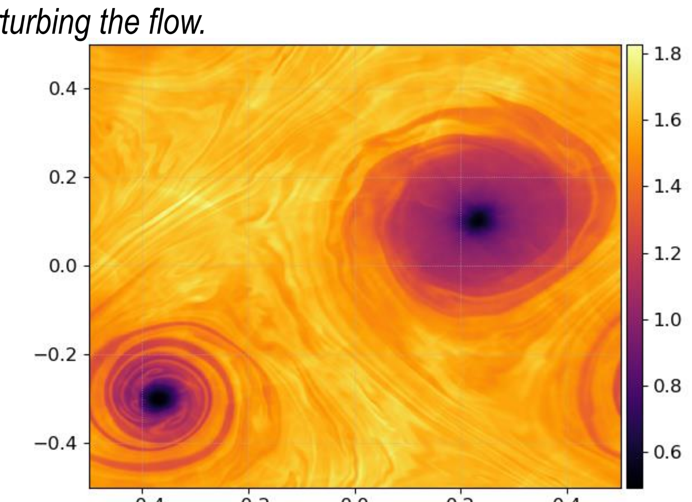


Figure 4: The dissipation region of the flow density depicted at time = 20. The vortices have interacted with one another to create two big vortices. Most of the energy in the fluid has dissipated.

- From *Figures 2-4,* one can observe many parameters changing as the flow goes through the three main stages of turbulence. In order to analyze the amount of power in the fluid, the energy power spectrum is plotted. *Table 1* below shows the resolution of each simulation.

Resolution	Number of Bins
192^2	136
292^2	207
392^2	278
492^2	348
592^2	419
692^2	490
792^2	560
892^2	631
992^2	702
1092^2	773
2092^2	1480

Table 1: The number of 'bins' or points given in the polar coordinates when the data was mapped. The number of bins was found by taking the resolution in one dimension and multiplying by the square root of 2.

- With the number of points in the polar spectrum, the energy power spectrum was calculated with varying resolution.

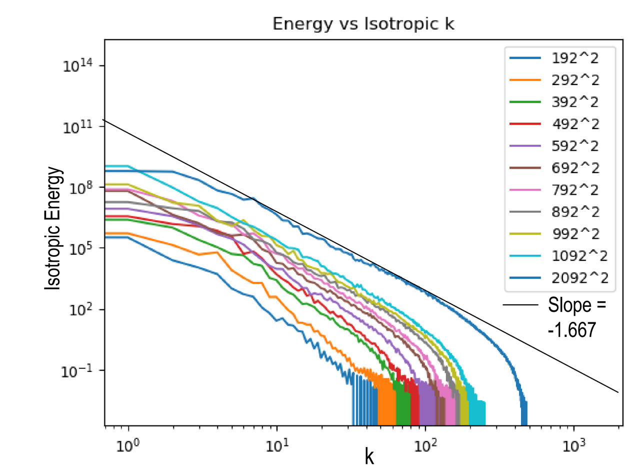


Figure 5: Energy power spectrum with varying resolutions is depicted. It has been observed that the power spectrum obeys Kolmogorov 1941 in the inertial sub-range, having a slope near -1.667. As resolution increases, the fluctuations in the data decreases because the grid becomes more refined.

Taylor-Green Vortex

- 3D simulation of an unsteady flow with a decaying vortex
- Simulation was initialized in the code with the following initial conditions:

$$V_0 = Ma \qquad \rho_0 = 1 \qquad t^* = V_0 t$$

$$v_x = V_0 \sin(x)\cos(y)\cos(z)$$

$$v_y = -V_0 \sin(y)\cos(x)\cos(z) \qquad a = \sqrt{\frac{\gamma p_0}{\rho_0}} \qquad \gamma = 1.4$$

$$v_z = 0 \qquad p_0 = 1 \qquad M = 0.1$$

$$p = p_0 + \frac{\rho_0 V_0^2}{16}[\cos(2x) + \cos(2y)][\cos(2z) + 2]$$

- The visualization of the z-vorticity is shown in *Figures 6-11*

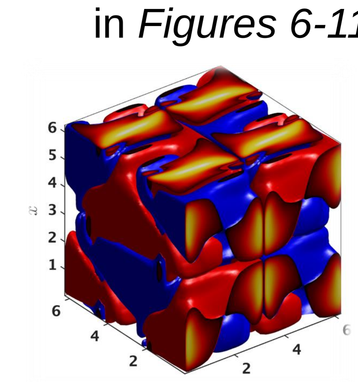


Figure 6: Simulation at t = 3, which shows inviscid flow.*

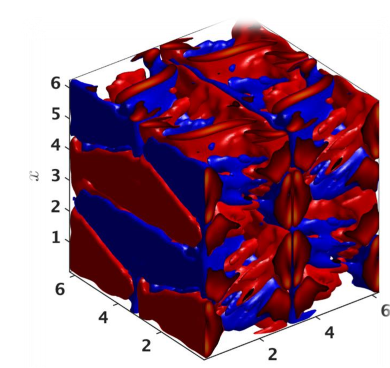


Figure 7: Simulation at t = 5.*

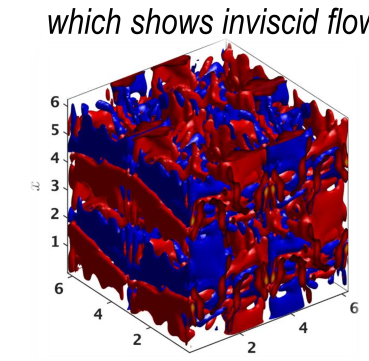


Figure 8: Simulation at t = 7.*

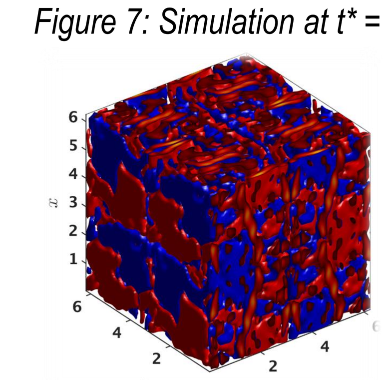


Figure 9: Simulation at t = 9.*

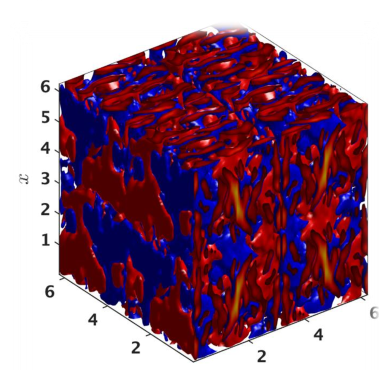


Figure 10: Simulation at t = 11, which shows fully turbulent flow.*

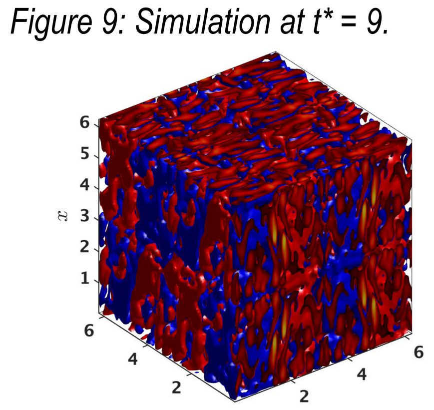
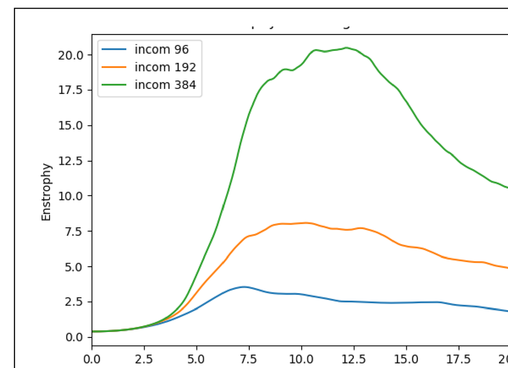


Figure 11: Simulation at t = 15, which shows turbulent decay.*

- This simulation can also be analyzed with enstrophy, rate of dissipation and generation of kinetic energy

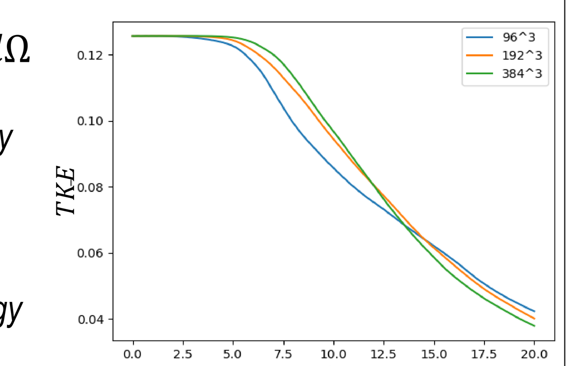


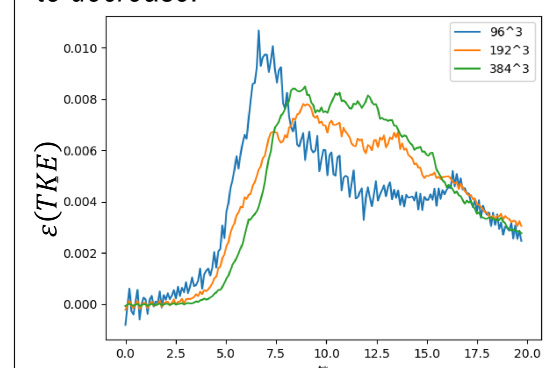
$$\varepsilon(\boldsymbol{u}) = \int_S |\nabla \times \boldsymbol{u}|^2 dS$$

Figure 12: Enstrophy plot with increasing resolution as a function of t. The peak occurs before t* = 10, which is in agreement with Figures 6-11.*

$$TKE = \frac{1}{\rho_0 \Omega}\int_\Omega \rho \frac{u_i u_i}{2} d\Omega$$

Figure 13: Total kinetic energy plotted with increasing resolution as a function of t. As the vortex decays, it is expected for the kinetic energy to decrease.*

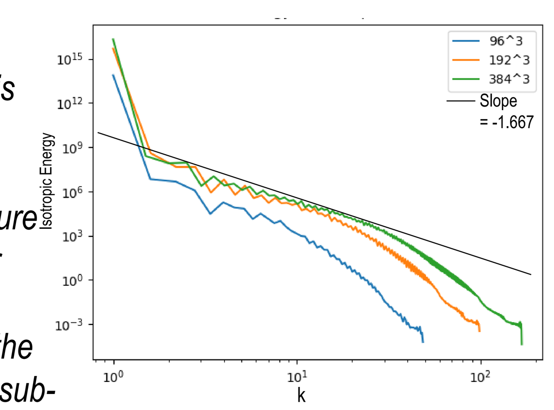




$$\varepsilon(TKE) = -\frac{d(TKE)}{dt}$$

Figure 15: The rate of dissipation of total kinetic energy plotted with increasing resolution as a function of t.*

Figure 16: The energy power spectra with increasing resolution. As the resolution is increased, the number of fluctuations within the data decreases. Compared to Figure 5, the spectra covers a lower range of values although the resolution in 3D is less than the resolution in 2D. The inertial sub-range has a slope of -1.667.



Orszag-Tang Vortex

- This 2D simulation is a subsonic plasma-fluid simulation, because it approximates the plasma as a fluid.
- The simulation follows the nonlinear evolution and decay of a vortex in an existing magnetic field.
- As the vortex decays, the magnetic field becomes twisted, inducing current sheets.
- These current sheets start to interact with one another, causing magnetic reconnection.

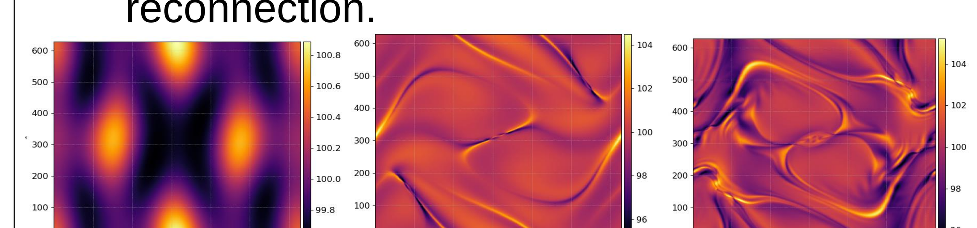


Figure 17: Shows the ion density t=0.1 (left figure), t=1.5 (middle figure), and at t=2.5 (right figure).

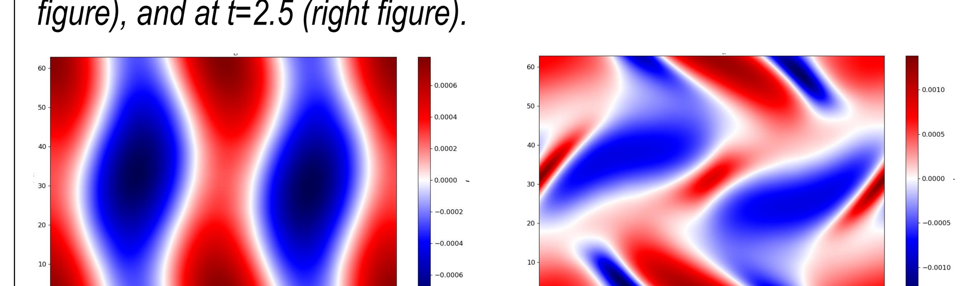


Figure 18: Shows the current density in the z-direction at t=0.1 (left figure) and at t=0.7 (right figure).

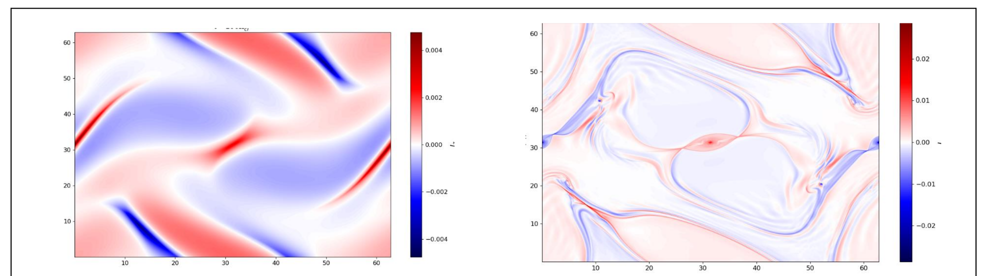


Figure 19: Depicts the current density in the z-direction at t=1 (left figure) and at t=2.5 (right figure).

Effect of Resolution on Turbulence

- In the 2D case of Kelvin-Helmholtz, increasing resolution resolves small scales and gives a longer inertial sub-range.

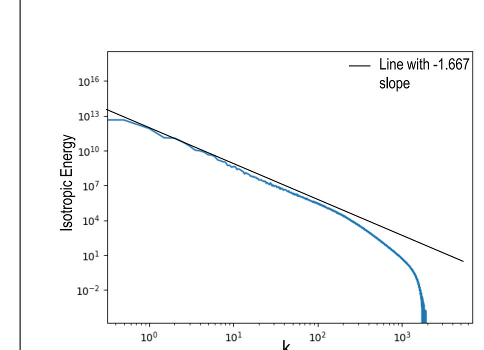


Figure 20: The energy power spectrum for a resolution of 12288^2. The spectrum is more refined than Figure 5, and the resolution does not have any affect on the energy production range. The inertial subrange is longer, with a slope of -1.6667.

- In the 3D case of Taylor-Green, increasing resolution not only resolves smaller scales, it also gives a larger amount of energy dissipation within the system (*Figure 12*).

Future Work

- Analyze more turbulence test cases to check if code gives correct results
- Analyze more turbulence test cases to observe if there are other commonalities among turbulence simulations
- Analyze the Orszag-Tang vortex in more detail to see why isotropic electric field spectra has a sharp initial decay.

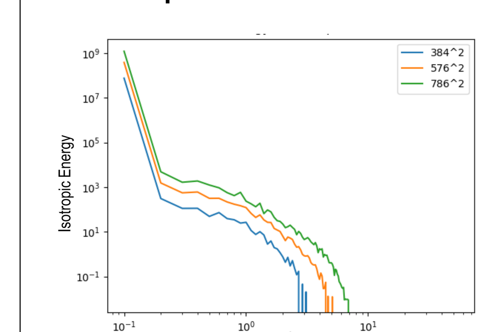


Figure 21: Depicts the isotropic electric field spectra for increasing resolution. It is currently not known why there is a sharp initial decay.

References

[1] Frisch, Uriel. *Turbulence: the Legacy of A.N. Kolmogorov*. Cambridge University Press, 1995.

[2] James, DeBonis R. "Solutions of the Taylor-Green Vortex Problem Using High-Resolution Explicit Finite Difference Methods." *Science and Technical Information Program at NASA*, 2013.

[3] *Figure 1* Image: https://w3.pppl.gov/~hammett/viz/viz.html

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